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Low-complexity robust adaptive generalized sidelobe canceller detector for DS/CDMA systems

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SUMMARY

A novel low computational complexity robust adaptive blind multiuser detector, based on the minimum output energy (MOE) detector with multiple constraints and a quadratic inequality (QI) constraint is developed in this paper. Quadratic constraint has been a widespread approach to improve robustness against mismatch errors, uncertainties in estimating the data covariance matrix, and random perturbations in detector parameters. A diagonal loading technique is compulsory to achieve the quadratic constraint where the diagonal loading level is adjusted to satisfy the constrained value. Integrating the quadratic constraint into recursive algorithms seems to be a moot point since there is no closed-form solution for the diagonal loading term. In this paper, the MOE detector of DS/CDMA system is implemented using a fast recursive steepest descent adaptive algorithm anchored in the generalized sidelobe canceller (GSC) structure with multiple constraints and a QI constraint on the adaptive portion of the GSC structure. The Lagrange multiplier method is exploited to solve the QI constraint. An optimal variable loading technique, which is capable of providing robustness against uncertainties and mismatch errors with low computational complexity is adopted. Simulations for several mismatch and random perturbations scenarios are conducted in a rich multipath environment with near–far effect to explore the robustness of the proposed detector.

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1. INTRODUCTION

Constrained optimization methods have received considerable attention as a means to derive blind multiuser receivers for DS/CDMA system with low complexity [1, 2]. Most of the constrained optimization approaches [3–6] are based on the well-known RLS algorithm. In the conventional RLS algorithm, the calculation of the Kalman gain requires matrix inversion of the received signal covariance matrix. When the data covariance matrix is in ill condition the conventional RLS algorithm will rapidly become impossible [7]. Furthermore, when the estimated data covariance matrix lacks the property of positive definiteness, the algorithm will diverge [8]. Moreover, the least-square detectors need more robustness against pointing errors and perturbations in detector parameters.

To overcome the above shortcomings, a robust low-complexity blind multiuser receiver is developed using a fast recursive steepest descent (RSD) algorithm based on the generalized sidelobe canceller (GSC) structure. The RSD algorithm is analogous to the recursive conjugate gradient (RCG) algorithm [8–10]. The RSD algorithm is employed to update the adaptive weight vector of the GSC structure with optimum step-size for rapid convergence [11, 12]. Furthermore, a low computational complexity recursive update formula for the gradient vector of RSD algorithm is derived.

Additionally, a quadratic inequality (QI) constraint on the weight vector norm is imposed to manage the residual signal mismatch and other random perturbations errors. Moreover, the QI constraint freezes the noise constituent in the output signal-to-interference-plus-noise ratio (SINR) during adaptive implementation and hence overcomes noise enhancement at low SNR. Quadratic constraints have been used in adaptive beamforming for a variety of purposes such as improving robustness against mismatch and modelling errors, controlling mainlobe response, and enhancing interference cancellation capability [13–18]. An eigendecomposition approach is proposed in [18] to solve the QI constraint, which requires $O(N^3)$ complexity where N is the weight vector length. In [16, 17], new techniques for integrating the QI constraint into RLS algorithm are developed. Moreover, closed-form solutions for the diagonal loading term with $O(N^2)$ complexity are provided. These approaches provide a robust linearly constraint minimum variance beamformer based on the RLS algorithm with a variable loading (VL) technique based on Taylor series expansion approximation. The approaches in [16, 17] are extended to the DS/CDMA multiuser detection problem in [6, 19], respectively. These approaches provided robust DS/CDMA receivers based on the RLS algorithm with a QI constraint on the weight vector norm. Unfortunately, the developed VL techniques in [6, 19] depend on an approximation that is valid for small loading level. Therefore, for large loading levels these techniques fail to acquire the optimum diagonal loading amount [16, 17]. In addition to this, the proposed recursive implementations are based on RLS [6, 16, 17] or inverse QRD-RLS [19] algorithms. Efficient VL implementations for the robust adaptive beamforming problem are introduced in [17, 20], which require $O(N)$ complexity. These VL techniques are embedded with gradient minimization algorithms to estimate the robust adaptive beamformer.

Motivated by the above discussion, an alternative way of robust recursive multiuser detector is presented in this paper (also see [11, 12]), based on the low-complexity RSD adaptive algorithm with a QI constraint on the weight vector norm. An accurate VL technique based on the GSC structure is developed for precisely computing the diagonal loading level without eigendecomposition or Taylor approximation. The VL technique is integrated into the RSD adaptive algorithm, which exploited to update the adaptive weight vector of the GSC structure. The geometrical interpretation of the RSD-based VL technique is demonstrated.

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The remainder of the paper is organized as follows. In Section 2, linear detection, GSC structure, and RSD algorithm are outlined. In Section 3, the proposed robust adaptive detector is developed and its convergence analysis and geometric representation are presented. Computer simulations and performance comparisons are carried out in Section 4. Conclusions and points for future work are summarized in Section 5.

2. BACKGROUND

A chip-rate linear detector is generally designed by collecting N_f samples from the received discrete-time signal $\mathbf{x}(n)$. The linear detector output is a linear combination of the received chip-sampled signals [3–7], i.e.

$$y(n)=\mathbf{f}^{\mathrm{H}}(n)\mathbf{x}(n) \tag{1}$$

where $\mathbf{f}^{\mathrm{H}}(n)$ is an $N_f\times 1$ vector consisting of the weights and $(\cdot)^{\mathrm{H}}$ stands for the Hermitian transpose. The detector output energy is given by [2, 6]

$$E\{|\mathbf{y}(n)|^2\}=E\{|\mathbf{f}^{\mathrm{H}}(n)\mathbf{x}(n)|^2\}=\mathbf{f}^{\mathrm{H}}(n)\mathbf{R}_{xx}(n)\mathbf{f}(n) \tag{2}$$

where the matrix $\mathbf{R}_{xx}(n)=E\{\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)\}$ is the received signal covariance matrix and $E\{\cdot\}$ stands for the expectation operator.

The minimum output energy (MOE) detector can be obtained by minimizing the output energy of the receiver subject to certain number of constraints. To avoid the cancellation of the signal of interest scattered in different multipaths during the minimization of the detector output energy, we can generally impose a set of linear constraints of the form $\mathbf{C}_1^{\mathrm{H}}\mathbf{f}=\mathbf{g}$ where $\mathbf{C}_1$ is $N_f\times N_g$ matrix consisting of shifted versions of the interested user signature waveform [2–7] and $\mathbf{g}$ is a $N_g\times 1$ vector of constraints.

Therefore, the optimal MOE detector can be obtained by solving the following constrained minimization problem:

$$\min_{\mathbf{f}} \mathbf{f}^{\mathrm{H}}\mathbf{R}_{xx}\mathbf{f} \quad \text{s.t.} \quad \mathbf{C}_1^{\mathrm{H}}\mathbf{f}=\mathbf{g} \tag{3}$$

The so-called GSC structure was originally implemented in adaptive array processing and beamforming algorithms [21–23]. The essence behind the GSC structure is to convert a *constrained* optimization problem to an *unconstrained* optimization problem. This is accomplished by dividing the detector vector into two parts: a non-adaptive $N_f\times 1$ vector $\mathbf{f}_c$, which satisfies the constraints, and an adaptive $(N_a=N_f-N_g)\times 1$ vector $\mathbf{f}_a$, which can be adapted without constraints using any adaptation algorithm up to a certain tolerance level. The weight vector is then divided into two orthogonal parts and can be expressed as

$$\mathbf{f}=\mathbf{f}_c-\mathbf{B}\mathbf{f}_a \tag{4}$$

A blocking matrix $\mathbf{B}$ with dimension $N_f\times N_a$ is inserted to ensure the orthogonality between the upper and lower branches of the GSC structure, which satisfies $\mathbf{B}^{\mathrm{H}}\mathbf{C}_1=\mathbf{0}$ and $\mathbf{B}^{\mathrm{H}}\mathbf{B}=\mathbf{I}$. Therefore, using (4) the constrained optimization problem given in (3) can be converted to unconstrained minimization problem. The optimal weight vector $\mathbf{f}_a$ can be found by substituting (4) into (3) and

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minimizing the receiver's output energy as follows:

$$\min_{\mathbf{f}_a} (\mathbf{f}_c - \mathbf{B}\mathbf{f}_a)^{\mathrm{H}} \mathbf{R}_{xx} (\mathbf{f}_c - \mathbf{B}\mathbf{f}_a) \tag{5}$$

2.1. Recursive steepest descent (RSD) algorithm

The method of RSD is employed to recursively update the optimal weight vector that minimizes the following Lagrangian cost function [11, 12, 17, 24]:

$$\Psi(\mathbf{f}_a) = \tfrac{1}{2} (\mathbf{f}_c - \mathbf{B}\mathbf{f}_a)^{\mathrm{H}} \mathbf{R}_{xx} (\mathbf{f}_c - \mathbf{B}\mathbf{f}_a) \tag{6}$$

Therefore, the adaptive weight vector $\mathbf{f}_a$ of the GSC structure can be updated as follows:

$$\mathbf{f}_a(n) = \mathbf{f}_a(n-1) - \mu \nabla_{\mathbf{f}_a}(n-1) \tag{7}$$

where μ is the step-size of the algorithm and $\nabla_{\mathbf{f}_a}(n-1)$ is the conjugate derivative of $\Psi_{\mathbf{f}_a}$ with respect to $\mathbf{f}_a^{\mathrm{H}}$ determined at $(n-1)$ snapshot. From (6), the gradient vector can be computed as

$$\nabla_{\mathbf{f}_a} = -\mathbf{B}^{\mathrm{H}} \mathbf{R}_{xx} \mathbf{f}_c + \mathbf{B}^{\mathrm{H}} \mathbf{R}_{xx} \mathbf{B} \mathbf{f}_a = -\mathbf{P}_B \mathbf{f} \tag{8}$$

Therefore, the recursive implementation of $\mathbf{f}_a(n)$ can be acquired by substituting (8) into (7)

$$\mathbf{f}_a(n) = \mathbf{f}_a(n-1) - \mu(\mathbf{R}_B \mathbf{f}_a(n-1) - \mathbf{p}_B) \tag{9}$$

where $\mathbf{R}_B = \mathbf{B}^{\mathrm{H}} \mathbf{R}_{xx} \mathbf{B}$, $\mathbf{p}_B = \mathbf{P}_B \mathbf{f}_c$, $\mathbf{P}_B = \mathbf{B}^{\mathrm{H}} \mathbf{R}_{xx}$, and $\mathbf{f}(n-1) = \mathbf{f}_c - \mathbf{B}\mathbf{f}_a(n-1)$.

The convergence speed of the RSD algorithm is affected by the step-size selection [25]. In fact, a variable step-size approach increases the convergence speed of the algorithm and reduces the algorithm sensitivity to step-size selection. Therefore, in order to obtain an expression for the optimum step-size, we plug (9) in (6), which yields [25]:

$$\Psi(\mathbf{f}_a) = \tfrac{1}{2} (\mathbf{f}(n-1) + \mu \mathbf{B} \nabla_{\mathbf{f}_a}(n-1))^{\mathrm{H}} \mathbf{R}_{xx} (\mathbf{f}(n-1) + \mu \mathbf{B} \nabla_{\mathbf{f}_a}(n-1)) \tag{10}$$

By incorporating the variable step-size $\mu(n)$ into the above equation instead of the fixed step-size and after some manipulations, we obtain

$$\Psi_{\mathbf{f}_a}(n) = \Psi_{\mathbf{f}_a}(n-1) + \mu(n) \nabla_{\mathbf{f}_a}^{\mathrm{H}}(n-1) \mathbf{P}_B \mathbf{f}(n-1) + 0.5 \mu^2(n) \nabla_{\mathbf{f}_a}^{\mathrm{H}}(n-1) \mathbf{R}_B \nabla_{\mathbf{f}_a}(n-1) \tag{11}$$

where $\Psi_{\mathbf{f}_a}(n)$ is $\Psi(\mathbf{f}_a)$ determined at n snapshot. Equation (11) shows that $\Psi_{\mathbf{f}_a}(n)$ is a quadratic function of $\mu(n)$ and hence it has a global minimum. Assuming that $\Psi_{\mathbf{f}_a}(n-1)$ is independent of $\mu(n)$, the optimum step-size can be obtained by differentiating (11) with respect to $\mu(n)$ and equating the result with zero, we have

$$\mu_{\mathrm{opt}}(n) = - \left. \frac{\alpha \nabla_{\mathbf{f}_a}^{\mathrm{H}}(n-1) \mathbf{P}_B \mathbf{f}(n-1)}{\nabla_{\mathbf{f}_a}^{\mathrm{H}}(n-1) \mathbf{R}_B \nabla_{\mathbf{f}_a}(n-1)} \right|_{\nabla_{\mathbf{f}_a} = -\mathbf{P}_B \mathbf{f}} \Rightarrow \frac{\alpha \| \nabla_{\mathbf{f}_a}(n-1) \|^2}{\nabla_{\mathbf{f}_a}^{\mathrm{H}}(n-1) \mathbf{R}_B \nabla_{\mathbf{f}_a}(n-1)} \tag{12}$$

where α is a positive constant added to improve the numerical stability of the algorithm.

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3. ROBUST DETECTOR DESIGN

The robust detector is obtained by applying the QI constraint on the adaptive weight portion of the GSC structure (i.e. $\mathbf{f}_a$) [11–16] and consequently the following constrained optimization problem is obtained:

$$\mathbf{f}_a = \min_{\mathbf{f}_a} (\mathbf{f}_c - \mathbf{B}\mathbf{f}_a)^{\mathrm{H}} \mathbf{R}_{xx} (\mathbf{f}_c - \mathbf{B}\mathbf{f}_a) \quad \text{s.t.} \;\; \mathbf{f}_a^{\mathrm{H}} \mathbf{f}_a \leqslant \beta^2 \tag{13}$$

where the constrained value $\beta^2 = \delta^2 - \mathbf{f}_c^{\mathrm{H}} \mathbf{f}_c$, $\delta^2 = t \|\mathbf{f}_c\|^2$, and the constant t is set to an appropriate value.

The method of Lagrange multipliers is invoked to solve this constrained minimization problem by forming the following real-valued Lagrangian function [12, 16]:

$$\Phi(\mathbf{f}_a, \lambda_0) = \tfrac{1}{2} (\mathbf{f}_c - \mathbf{B}\mathbf{f}_a)^{\mathrm{H}} \mathbf{R}_{xx} (\mathbf{f}_c - \mathbf{B}\mathbf{f}_a) + \lambda_0 S(\mathbf{f}_a^{\mathrm{H}} \mathbf{f}_a - \beta^2) \tag{14}$$

where $s(\cdot)$ is a step function inserted to ensure that $\mathbf{f}_a^{\mathrm{H}} \mathbf{f}_a \geqslant \beta^2$ and the Lagrange multiplier λ_0 is a real scalar, which is determined from the QI constrained value. It should be non-negative to ensure that the diagonally loaded covariance data matrix is positive definite. The problem is then converted from *constrained* minimization to *unconstrained* minimization problem. The optimal solution is obtained by first taking the gradient of $\Phi(\mathbf{f}_a, \lambda_0)$ with respect to $\mathbf{f}_a^{\mathrm{H}}$ and then equating the resulting quantities to zero, we get,

$$\mathbf{f}_{a(\mathrm{opt})} = (\mathbf{R}_B + \lambda_0 \mathbf{I})^{-1} \mathbf{p}_B \tag{15}$$

The recursive implementation of the above detector incorporates several difficulties. In addition to the complicity due to the inverse matrix computation, the value of diagonal loading term λ_0 cannot be easily determined from the constrained value β^2 where there is no closed-form expression for the optimum loading level.

Recently, Tian *et al.* [6, 16] developed a VL technique based on Taylor series approximation and they adaptively incorporated it into the RLS algorithm. This VL technique relies on the approximation, which is generally valid for small loading level. Therefore, for large loading levels this technique fails to reach the optimum loading amount. The loading required at certain step in the VL manner is an *incremental* loading level that supplements the loading incorporated at all the previous steps. This is usually a small amount and the optimal incremental loading level to satisfy the quadratic constraint can usually be achieved. However, in some dynamic scenarios, abrupt mismatch may lead to performance degradation due to the requirement of large diagonal loading level, which cannot be achieved by this technique. In addition to this, the diagonal loading term λ_0 requires $O(N_a^2 + 2N_a)$ multiplications.

Being motivated by the above discussion, a new optimal VL technique capable of precisely computing the optimum diagonal loading term λ_0 is developed in this paper. In addition, the proposed VL technique is efficiently integrated into the RSD algorithm (referred to as RSD-VL) for recursively estimating the optimal robust detector with $O(N_a)$ complexity. The value of the optimum diagonal loading level is estimated without approximation. The RSD algorithm is exploited to recursively update the optimal weight vector that minimizes (14), i.e.

$$\Phi(\tilde{\mathbf{f}}_a, \lambda_0) = \Psi(\tilde{\mathbf{f}}_a) + \lambda_0 s(\tilde{\mathbf{f}}_a^{\mathrm{H}} \tilde{\mathbf{f}}_a - \beta^2) \tag{16}$$

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where $\tilde{\mathbf{f}}_a$ stands for the robust detector. As a consequence,

$$\tilde{\mathbf{f}}_a(n) = \tilde{\mathbf{f}}_a(n-1) - \mu(n)\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1) \tag{17}$$

where

$$\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1) = \nabla_{\mathbf{f}_a}(n-1) + \lambda_0(n)\tilde{\mathbf{f}}_a(n-1) \tag{18}$$

and $\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1)$ stands for the robust gradient and $\nabla_{\mathbf{f}_a}(n-1)$ stands for the non-robust gradient.

Then, the robust detector can be expressed as follows:

$$\tilde{\mathbf{f}}_a(n) = \mathbf{f}_a(n) - \mu(n)\lambda_0(n)\tilde{\mathbf{f}}_a(n-1) \tag{19}$$

where

$$\mathbf{f}_a(n) = \tilde{\mathbf{f}}_a(n-1) - \mu(n)\nabla_{\mathbf{f}_a}(n-1) \tag{20}$$

The step function that is dropped as the second term in (21) is incorporated only when $\mathbf{f}_a^{\mathrm{H}}(n)\mathbf{f}_a(n) > \beta^2$. The QI constraint should be satisfied at each iteration step, i.e. $\tilde{\mathbf{f}}_a^{\mathrm{H}}(n)\tilde{\mathbf{f}}_a(n) \leqslant \beta^2$, and assuming the constraint was satisfied in the previous step and using (19), we have

$$(\mathbf{f}_a(n) - \mu(n)\lambda_0(n)\tilde{\mathbf{f}}_a(n-1))^{\mathrm{H}}(\mathbf{f}_a(n) - \mu(n)\lambda_0(n)\tilde{\mathbf{f}}_a(n-1)) \leqslant \beta^2 \tag{21}$$

The value of λ_0 can be found by solving the following quadratic equation (only if the QI constraint is not met):

$$\mu(n)^2\|\tilde{\mathbf{f}}_a(n-1)\|^2\lambda_0(n)^2 - 2\mu(n)\mathrm{Re}\{\mathbf{f}_a^{\mathrm{H}}(n)\tilde{\mathbf{f}}_a(n-1)\}\lambda_0(n) + \mathbf{f}_a^{\mathrm{H}}(n)\mathbf{f}_a(n) - \beta^2 = 0 \tag{22}$$

Therefore, the value of $\lambda_0(n)$, which satisfies the QI constraint is obtained by solving (22), which yields:

$$\begin{aligned} \lambda_0(n) &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ a &= \mu(n)^2\|\tilde{\mathbf{f}}_a(n-1)\|^2 \end{aligned} \tag{23}$$

where

$$\begin{aligned} b &= -2\mu(n)\mathrm{Re}\{\mathbf{f}_a^{\mathrm{H}}(n)\tilde{\mathbf{f}}_a(n-1)\} \\ c &= \|\mathbf{f}_a(n)\|^2 - \beta^2 \end{aligned} \tag{24}$$

When the QI constraint is not satisfied (i.e. $\mathbf{f}_a^{\mathrm{H}}(n)\mathbf{f}_a(n) > \beta^2$) and consequently $C > 0$, the roots of the quadratic equation (22) are either two real-positive values or a conjugate pair whose real parts are positive. Therefore, the condition that λ_0 is positive is satisfied. Furthermore, the complex roots can be avoided by properly selecting the step-size, which cannot be guaranteed by the VL technique presented in [16]. Moreover, the proposed VL technique depends only on the previous update of the weight vector as shown in (19). Consequently, the total amount of the required multiplications to calculate the diagonal loading term $\lambda_0(n)$ is about $O(2N_a)$. Compared with the VL technique in [16], the new VL technique has a substantially lower computational complexity.

We should stress on the difference between the proposed RSD-VL technique developed in this paper and the simple VL least mean square (LMS) algorithm. It is demonstrated in [16, 24] that

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the VL LMS technique does not offer additional improvement over the scaled projection LMS approach or even over the LMS algorithm without the QI constraint [12].

It is important to derive the sufficient conditions on the step-size, which guarantee real-positive roots for (22). The following two conditions state the necessary and sufficient conditions under which (22) has real and positive solutions, respectively:

Condition 1 ($b^2-4ac\geqslant 0$): By substituting (20) and (24) into $b^2-4ac\geqslant 0$ and similar to the approach in [17], the following inequality is obtained:

$$\mu(n)\leqslant\frac{\beta\|\tilde{\mathbf{f}}_a(n-1)\|}{\sqrt{\|\tilde{\mathbf{f}}_a(n-1)\|^2\|\nabla_{\mathbf{f}_a}(n-1)\|^2-\nabla^{\mathrm{H}}_{\mathbf{f}_a}(n-1)\tilde{\mathbf{f}}_a(n-1)\tilde{\mathbf{f}}^{\mathrm{H}}_a(n-1)\nabla_{\mathbf{f}_a}(n-1)}} \tag{25}$$

The above inequality is valid for real vectors. However, it can be generalized to complex vectors as in [17]. Therefore, this upper bound on $\mu(n)$ guarantees real-positive roots for (22) and consequently, the optimum loading level can be obtained. The equality in (25) represents the one positive real-root solution of (22), i.e. $b^2-4ac=0\Rightarrow\lambda_0=\mathrm{Re}\{\mathbf{f}^{\mathrm{H}}_a(n)\tilde{\mathbf{f}}_a(n-1)\}/\mu(n)\|\tilde{\mathbf{f}}_a(n-1)\|^2$.

Additionally, based on the well-known Cauchy-Schwarz inequality [26] and the fact that $\tilde{\mathbf{f}}^{\mathrm{H}}_a(n-1)\tilde{\mathbf{f}}_a(n-1)\leqslant\beta^2$, it is easy to verify that

$$\|\tilde{\mathbf{f}}_a(n-1)\|^2\|\nabla_{\mathbf{f}_a}(n-1)\|^2\geqslant\nabla^{\mathrm{H}}_{\mathbf{f}_a}(n-1)\tilde{\mathbf{f}}_a(n-1)(\nabla^{\mathrm{H}}_{\mathbf{f}_a}(n-1)\tilde{\mathbf{f}}_a(n-1))^{\mathrm{H}} \tag{26}$$

Therefore, this inequality guarantees the existence of the step-size upper bound for any gradient vector $\nabla_{\mathbf{f}_a}(n-1)$. The equality in (26) occurs when the $\nabla_{\mathbf{f}_a}(n-1)$ vector has the same direction of the detector $\tilde{\mathbf{f}}_a(n-1)$.

Condition 2 ($b\leqslant 0$): In addition to (25), one more constraint on the step-size is obligatory to guarantee positive diagonal loading (i.e. $b\leqslant 0$). Substituting (9) and (20) into $b\leqslant 0$, yields:

$$\mu(n)[\tilde{\mathbf{f}}_a(n-1)^{\mathrm{H}}\mathbf{R}_B(n)\tilde{\mathbf{f}}_a(n-1)-\tilde{\mathbf{f}}_a(n-1)^{\mathrm{H}}\mathbf{P}_B(n)\mathbf{f}_c]\leqslant\|\tilde{\mathbf{f}}_a(n-1)\|^2 \tag{27}$$

The sign on the left-hand side of (27) is determined based on the constrained value $\beta^2=(t-1)\|\mathbf{f}_c\|^2$. In several practical scenarios, the constrained parameter t is selected within $(1,2]$. Therefore, the sign on the left-hand side is negative and hence positive diagonal loading (i.e. $b\leqslant 0$) is always guaranteed.

3.1. Low-complexity recursive implementation

Summarizing the proposed adaptive receiver (referred to as MOE-RSD w. QC), the matrices $\mathbf{R}_B(n)$ and $\mathbf{P}_B(n)$ are updated using exponentially decaying data windows as follows:

$$\mathbf{R}_B(n)=\eta\mathbf{R}_B(n-1)+\mathbf{z}(n)\mathbf{z}^{\mathrm{H}}(n) \tag{28}$$

$$\mathbf{P}_B(n)=\eta\mathbf{P}_B(n-1)+\mathbf{z}(n)\mathbf{x}^{\mathrm{H}}(n) \tag{29}$$

where

$$\mathbf{z}(n)=B^{\mathrm{H}}\mathbf{x}(n) \tag{30}$$

and η is the usual forgetting factor with $0<<\eta\leqslant 1$. The unconstrained detector $\mathbf{f}_a(n)$ is updated using (20). When $\mathbf{f}_a(n)$ does not fulfil the QI constraint, the optimum loading level is calculated using (23) and the robust detector $\tilde{\mathbf{f}}_a(n)$ is computed using (19).

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In spite of the simplicity of the proposed robust MOE-RSD w. QC algorithm, its recursive implementation requires high computational load resulting from storing and updating the two matrices $\mathbf{R}_B$ and $\mathbf{P}_B$. To overcome this drawback, the gradient vector can be recursively updated by substituting (9), (17)–(20), (28), and (29) into (8) and after some manipulations, the following recursive update equation for gradient vector is obtained:

$$\nabla_{\mathbf{f}_a}(n) = \eta\nabla_{\mathbf{f}_a}(n-1) - y(n)\mathbf{z}(n) - \mu(n)\mathbf{R}_B(n)\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1) \quad (31)$$

where

$$y(n) = y_c(n) - y_a(n) = \mathbf{x}^{\mathrm{H}}(n)\mathbf{f}_c - \mathbf{z}^{\mathrm{H}}(n)\tilde{\mathbf{f}}_a(n-1) \quad (32)$$

Therefore, the matrix $\mathbf{R}_B(n)$ is merely updated. The second gradient vector on the right-hand side of (31) $\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1)$ includes pervious diagonal loading effects as shown in (18).

The proposed robust MOE-RSD w. QC algorithm with the required amount of multiplications at each step is summarized in Table I. An *if–else* condition statement is added to ensure that $C>0$; if this condition is achieved, the VL subroutine is executed and a new update procedure for the detector is conducted based on (19) and (23). Otherwise, the algorithm resumes without executing the VL subroutine. Equation (23) contains two roots; the smaller root is selected to guarantee stability of the algorithm. As shown in Table I, the total amount of the required multiplications at each snapshot of the proposed detector is about $O(2N_a^2 + N_aN_f + 6N_a + N_f)$, including $\mathbf{R}_B(n)$ update, the VL technique, optimum step-size estimation, and the robust detector update, while the robust detector in [6] requires about $O(3N_a^2 + N_aN_f + 6N_a + N_f)$ multiplications. The non-robust version of the proposed detector (referred to as MOE-RSD) can be obtained by removing the VL technique from Table I.

3.2. *Convergence analysis*

To examine the convergence of the proposed algorithm, we will derive conditions on the step-size, which guarantee that the detector $\mathbf{f}_a(n)$ asymptotically converges to its optimal value given in (15). This can be done by evaluating the difference between the expected value of the recursive detector $\mathbf{f}_a(n)$ and its optimal value [15], i.e.

$$\varepsilon_{\mathbf{f}_a}(n) = E\{\tilde{\mathbf{f}}_a(n)\} - \mathbf{f}_{a(\mathrm{opt})} \quad (33)$$

By substituting (9), (15), and (19) into (33), we get

$$\varepsilon_{\mathbf{f}_a}(n) = (I - \mu(n)(\mathbf{R}_B + \lambda_0(n)\mathbf{I}))E\{\tilde{\mathbf{f}}_a(n-1)\} + \mu(n)\mathbf{p}_B - \mathbf{f}_{a(\mathrm{opt})} \quad (34)$$

After some manipulations, we can write (34) as follows:

$$\varepsilon_{\mathbf{f}_a}(n) = (\mathbf{I} - \mu(n)(\mathbf{R}_B + \lambda_0(n)\mathbf{I}))\varepsilon_{\mathbf{f}_a}(n-1) + \mu(n)\mathbf{p}_B - \mu(n)(\mathbf{R}_B + \lambda_0(n)\mathbf{I})\mathbf{f}_{a(\mathrm{opt})} \quad (35)$$

It can be seen that the last two terms are cancelled, and hence a recursive formula for $\varepsilon_{\mathbf{f}_a}$ is obtained as follows:

$$\varepsilon_{\mathbf{f}_a}(n) = (\mathbf{I} - \mu(n)(\mathbf{R}_B + \lambda_0(n)\mathbf{I}))\varepsilon_{\mathbf{f}_a}(n-1) \quad (36)$$

Therefore, the convergence of the algorithm depends on the eigenvalue spread of the diagonally loaded blocked data matrix. If the eigenvalues of $(\mathbf{R}_B + \lambda_0\mathbf{I})$ are denoted by ρ_i, then the proposed

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Table I. Summary of the robust MOE-RSD w. QC detector.

Initialization

$\mathbf{R}_B(0)=\mathbf{B}^{\mathrm{H}}(\zeta\mathbf{I}_{N_a})\mathbf{B},\ \mathbf{f}_c=\mathbf{C}_1(\mathbf{C}_1^{\mathrm{H}}\mathbf{C}_1)^{-1}\mathbf{g}$

$\delta^2=t\cdot\|\mathbf{f}_c\|^2,\ \beta^2=\delta^2-\|\mathbf{f}_c\|^2$

$\alpha=0.1,\ \lambda_0^1(0)=0$

$\mathbf{f}_a(n)=\mathbf{0}_{N_a\times 1},\ \bar{\nabla}_{\tilde{\mathbf{f}}_a}(0)=\mathbf{B}^{\mathrm{H}}(\zeta\mathbf{I}_{N_a})\mathbf{f}_c$

For $n=1,2,\ldots,$ do

$\mathbf{z}(n)=\mathbf{B}^{\mathrm{H}}\mathbf{x}(n);\ N_a N_f$

$y(n)=\mathbf{x}^{\mathrm{H}}(n)\mathbf{f}_c+\mathbf{z}^{\mathrm{H}}(n)\tilde{\mathbf{f}}_a(n-1);\ N_f+N_a$

$\mathbf{R}_B(n)=\eta\mathbf{R}_B(n-1)+\mathbf{z}^{\mathrm{H}}(n)\mathbf{z}(n),\ N_a^2$

$\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1)=\nabla_{\mathbf{f}_a}(n-1)+\lambda_0^1(n-1)\tilde{\mathbf{f}}_a(n-1)$

$$\mu(n)=\frac{\alpha\|\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1)\|^2}{\bar{\nabla}^{\mathrm{H}}_{\tilde{\mathbf{f}}_a}(n-1)\mathbf{R}_B(n)\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1)};\quad N_a+N_a^2$$

$\nabla_{\mathbf{f}_a}(n)=\eta\nabla_{\mathbf{f}_a}(n-1)-y(n)\mathbf{z}(n)-\mu(n)\mathbf{R}_B(n)\bar{\nabla}_{\tilde{\mathbf{f}}_a}(n-1)$

$\mathbf{f}_a(n)=\tilde{\mathbf{f}}_a(n-1)-\mu(n)\nabla_{\mathbf{f}_a}(n)$

If $(\|\mathbf{f}_a(n)\|^2>\beta^2);\ N_a$

$a=\mu(n)^2\|\tilde{\mathbf{f}}_a(n-1)\|^2;\ N_a$

$b=-2\mu(n)\mathrm{Re}\{\mathbf{f}_a^{\mathrm{H}}(n)\tilde{\mathbf{f}}_a(n-1)\},\ N_a$

$c=\|\mathbf{f}_a(n)\|^2-\beta^2$

$$\lambda_0^1(n)=\frac{-b-(\sqrt{b^2-4ac})}{2a}$$

$\tilde{\mathbf{f}}_a(n)=\mathbf{f}_a(n)-\mu(n)\lambda_0^1(n)\tilde{\mathbf{f}}_a(n-1)$

Else

$\lambda_0^1(n)=0$

$\tilde{\mathbf{f}}_a(n)=\mathbf{f}_a(n)$

End if

$\tilde{\mathbf{f}}(n)=\mathbf{f}_c-\mathbf{B}\tilde{\mathbf{f}}_a(n)$; (only for evaluation)

End for

End

robust detector is guaranteed to converge to its optimum value with optimum diagonal loading computation if the following general upper bound on the step-size is guaranteed:

$$0<\mu(n)<\min\left\{\frac{1}{\rho_{\max}},\frac{\beta\|\tilde{\mathbf{f}}_a(n-1)\|}{\sqrt{\|\tilde{\mathbf{f}}_a(n-1)\|^2\|\nabla_{\mathbf{f}_a}(n-1)\|^2-\nabla^{\mathrm{H}}_{\mathbf{f}_a}(n-1)\tilde{\mathbf{f}}_a(n-1)\tilde{\mathbf{f}}_a^{\mathrm{H}}(n-1)\nabla_{\mathbf{f}_a}(n-1)}}\right\}\quad(37)$$

The step-size inequality constraint in (37) could be used to oversee the update process of $\mathbf{f}_a(n)$ (*in advance*) to precisely compute the optimum diagonal loading value. Furthermore, several numerical simulations indicate that the optimum step-size, which guarantees faster convergence is always lower than the upper bound in (37) by an order of magnitude.

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3.3. Geometric interpretation

To examine the proposed RSD-VL technique, let us consider a simple 2D case. Figure 1 represents the proposed RSD-VL technique. Suppose that the vector $\mathbf{OA}=\tilde{\mathbf{f}}_a(n-1)$ fulfils the QI constraint, which is determined by the circle in the figure. After the next update and without the QI constraint, we obtain the vector $\mathbf{OB}=\mathbf{f}_a(n)$, which may not satisfy the QI constraint. The proposed VL technique is then invoked by adding the vector $\mathbf{BC1}=-\mu(n)\lambda_0^1\tilde{\mathbf{f}}_a(n-1)$ to the vector **OB**. As a consequence, the constrained vector **OC1**, which satisfies the QI constraint is obtained. The diagonal loading value $\lambda_0^1(n)$ represents the smaller root of (22) and $\lambda_0^2(n)$ represents the larger root. If the larger root is selected, a new constrained vector $\mathbf{OC2}=\overset{=}{\mathbf{f}}_a(n)$ is obtained. The smaller root $\lambda_0^1(n)$ is always preferred to guarantee algorithm stability.

An important observation is that the QI constrained region is a closed circle centred at the origin O and the concentric ellipses represent the unconstrained cost function in (6) and its centre is the minimal point without the QI constraint. Hence, the tangency point $C1$ with the QI constrained boundary is the optimum loading point in the sense of satisfying the QI constraint and the unconstrained MOE cost function (6).

3.4. Constrained value selection

The selection of the constrained value is a compromise among robustness, optimality, and computational load of the algorithm. Low constrained value leads to more robust algorithm at the expense of optimality and computational load. Increasing the constrained value decreases the number of VL subroutine execution, and hence decreases the computational load until a certain limit where

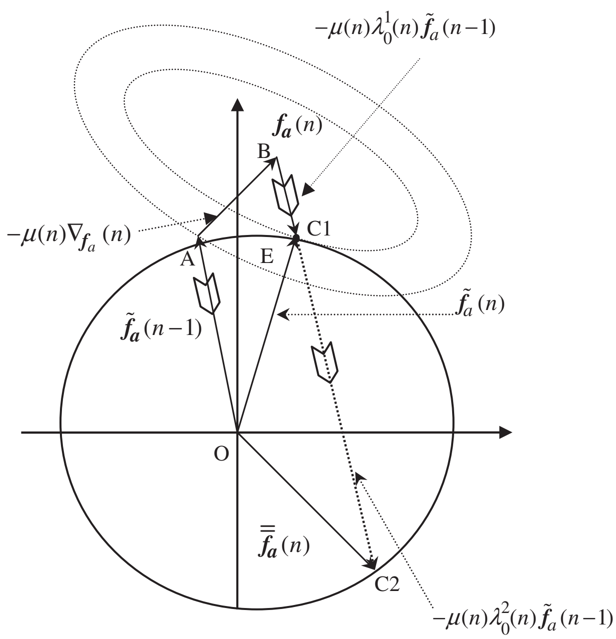

Figure 1. Geometric illustration of the proposed RSD-VL technique.

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the algorithm robustness starts to decline. The essence behind any QI constraint technique is to boost the robustness of the algorithm without affecting the optimality of the algorithm, which can be achieved without the existence of uncertainties and mismatch errors that necessitates adding robustness. Therefore, the constrained value can be set within a certain range of values. The reasonable range of the constrained value is not a static range but it is a dynamic range depending on the system parameters, the required amount of robustness, and the algorithm optimality. The appropriate value of the constrained value could be selected practically based on some preliminary (coarse) knowledge about wireless channels or using Monte Carlo simulation.

4. SIMULATIONS RESULTS

In this section, the performance of the proposed detector is investigated and compared with the traditional blind MOE detector, which updated using the RLS algorithm (referred to as MOE-RLS) and the robust detector proposed in [6] (referred to as MOE-RLS w. QC). Five synchronous users using 31 chips Gold codes in a multipath Rayleigh fading channel with 5 multipath components are simulated. The channel length (max delay spread) is assumed to be 10 delayed components and multipath delays are randomly distributed. The detector is assumed to be synchronized to the interested user. Users are assumed to have equal power except the required user, which is assumed to have 10 dB lower than other users and SNR is 30 dB. The detector length is assumed to span a single symbol interval. The output SINR and the bit error rate (BER) are adopted as the performance measures. The BER rate is determined by the receiver output SNR and the Euclidean distance between the receiver output and the decision boundary.

In order to verify the robustness of the proposed detector, five scenarios are analyzed. The first scenario is referred to as the ideal scenario, where all the initialization and algorithm parameters are properly selected. The initialization parameters of this scenario are as follows: $\eta = 1$, $\delta^2 = 1.35(\|\mathbf{f}_c\|^2)$, $\mathbf{f}_a(n) = \mathbf{0}_{N_a}$, and $\zeta = 1$. The selection of the constrained value is based on a

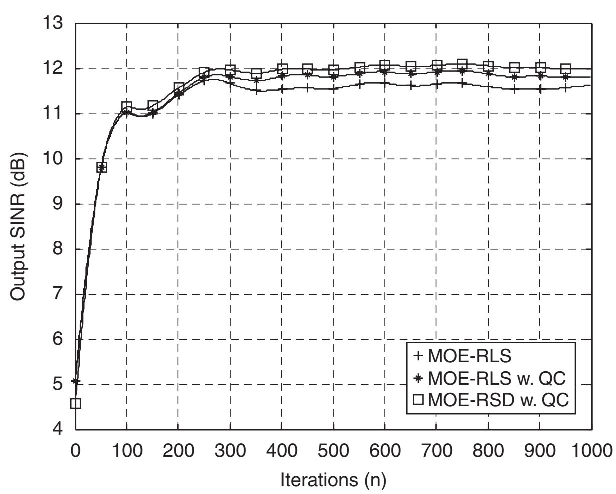

Figure 2. SINR versus iterations for the ideal scenario.

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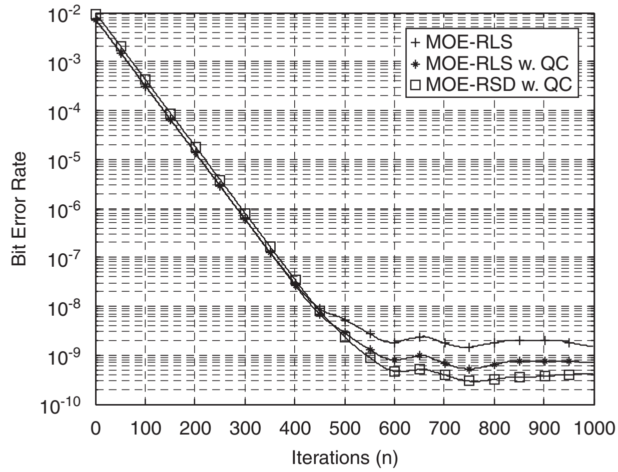

Figure 3. BER versus iterations for the ideal scenario.

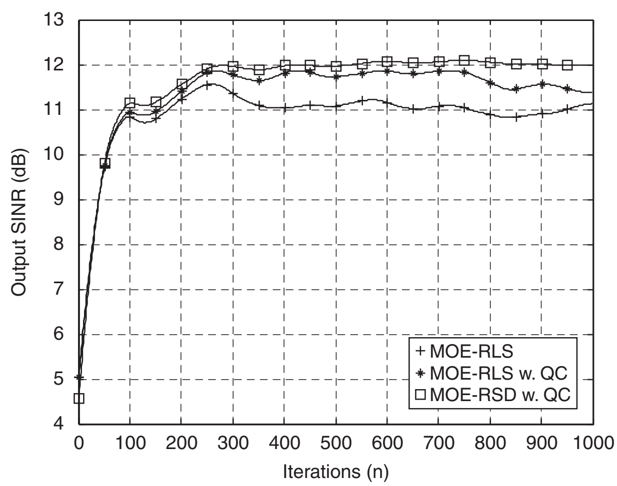

Figure 4. SINR versus iterations for the second scenario.

compromise between the robustness and the performance optimality. Figures 2 and 3 illustrate, respectively, the SINR and BER for the three detectors using the ideal scenario. The performance of the three detectors in terms of the convergence speed is almost the same where the ideal scenario decreases the necessity for robustness. Therefore, the VL subroutine is infrequently executed. Note however that, the figures show that the proposed detector offers little steady-state performance improvement over the MOE-RLS w. QC detector.

The second scenario is based on adding an uncertainty to the data covariance matrix by setting the forgetting factor to a smaller value, which is equivalent to a small sample support scenario. Figures 4 and 5 show, respectively, the SINR and BER for the three detectors with $\eta = 0.996$. The

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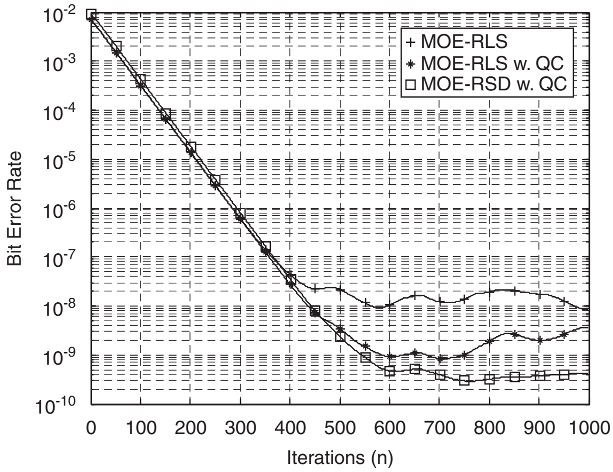

Figure 5. BER versus iterations for the second scenario.

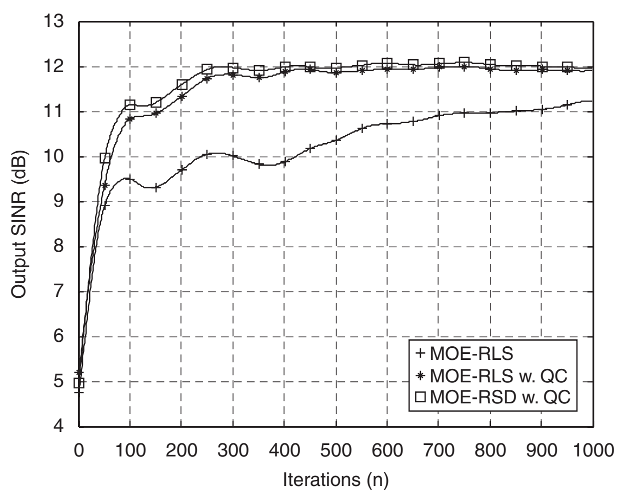

Figure 6. SINR versus iterations for the third scenario.

optimality of the proposed detector in terms of the robustness and the steady-state performance is evident from the figures.

The third scenario is constructed by improper initialization of the covariance matrix by setting $\zeta = 10$. Figures 6 and 7 demonstrate the SINR and BER for this scenario. In the fourth scenario, the detector parameters are improperly initialized such as $\mathbf{f}_a(0) = [1 \ \ldots \ 1]^{\mathrm{T}}$. The SINR and BER for this scenario are illustrated in Figures 8 and 9, respectively. The performance of the non-robust MOE-RLS detector is seriously degraded with these scenarios while the MOE-RSD w. QC detector is shown to have a comparable or a slightly better performance over the MOE-RLS w. QC detector.

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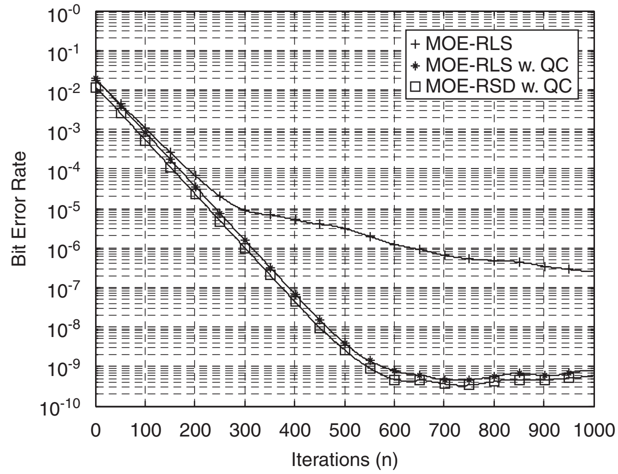

Figure 7. BER versus iterations for the third scenario.

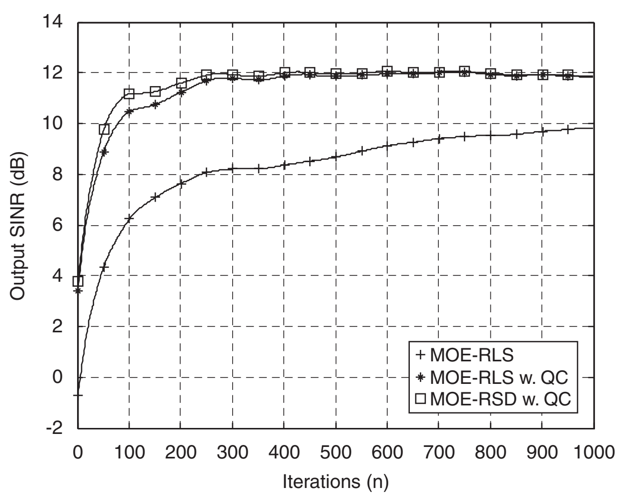

Figure 8. SINR versus iterations for the fourth scenario.

In the fifth scenario the SNR is reduced to 10 dB with the parameters of the ideal scenario. Figures 10 and 11 illustrate the performance of the three aforementioned detectors at low SNR scenario. In addition, the non-robust MOE-RSD detector is analyzed in this scenario. It is apparent from the figures that the quadratic constraint boosts the steady-state performance of the RLS and RSD algorithms. In addition, the MOE-RSD detector outperforms the MOE-RLS algorithm in terms of the steady-state performance and convergence speed.

We can observe from all scenarios that the proposed MOE-RSD w. QC detector outperforms the MOE-RLS w. QC detector and offers considerable improvement over the MOE-RLS algorithm.

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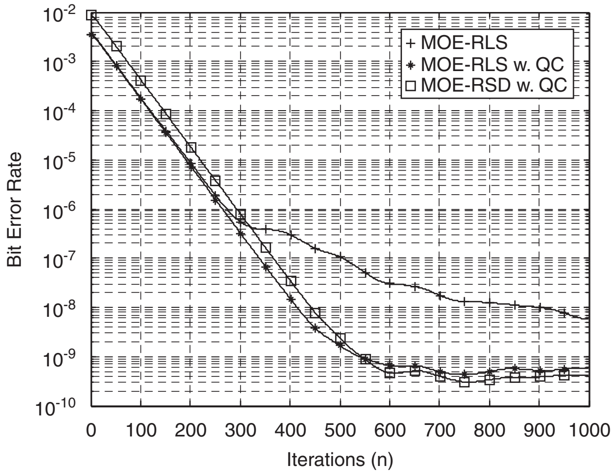

Figure 9. BER versus iterations for the fourth scenario.

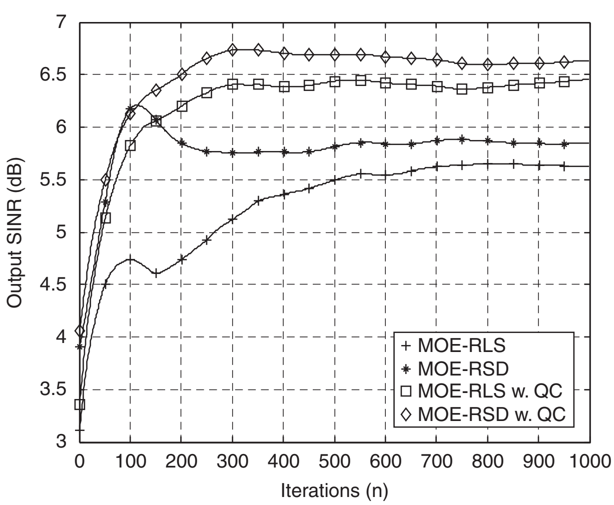

Figure 10. SINR versus iterations for the fifth scenario.

Additionally, the robustness of the quadratically constrained class of detectors compared with the traditional MOE detector is evident from simulations.

Finally, it is interesting to investigate the effect of the step-size on the convergence of the proposed adaptive detector. It is notable to emphasize the effect of the QI constraint on the MOE-RSD algorithm where it acts as a compensator for the improper step-size selection. Therefore, the QI constraint algorithm is less sensitive to the step-size selection. In order to corroborate the preceding finding, a new simulation is conducted in Figures 12 and 13 based on the fifth scenario. This simulation analyzes the effect of the improper step-size selection on the algorithm

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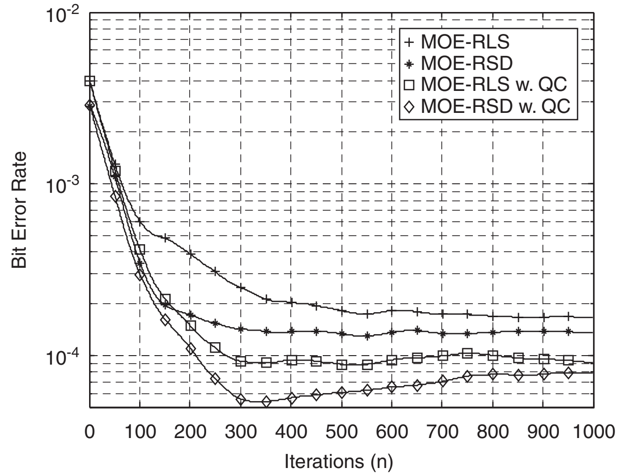

Figure 11. BER versus iterations for the fifth scenario.

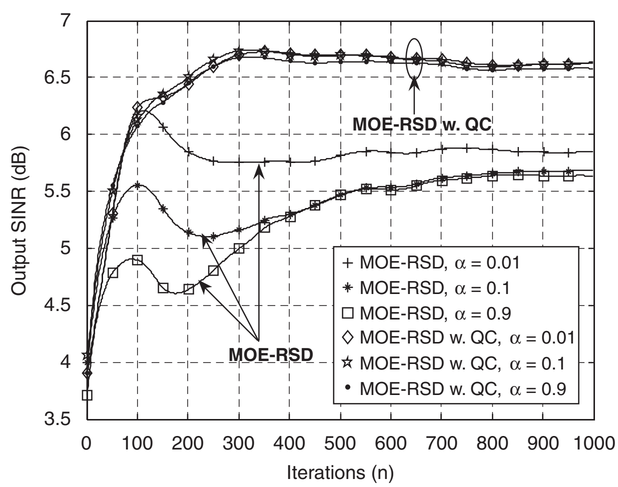

Figure 12. SINR for MOE-RSD and MOE-RSD w. QC algorithms with different α values.

performance. The improper step-size is modelled by the improper selection of the parameter α in (12). Three values for α are tested with both MOE-RSD and MOE-RSD w. QC detectors. The figures indicate that the QI constraint compensates the improper selection of the factor α. On the other hand, the steady-state performance and convergence rate of the non-robust MOE-RSD algorithm are affected by the selection of this factor. However, by comparing Figures 12 and 13 with Figures 10 and 11, the performance of the worst MOE-RSD algorithm ($\alpha=0.9$) is analogous to the MOE-RLS algorithm.

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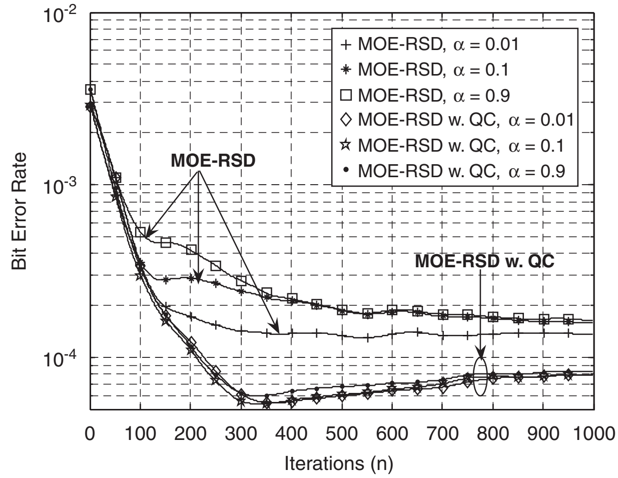

Figure 13. BER for MOE-RSD and MOE-RSD w. QC algorithms with different α values.

5. CONCLUSIONS

In this paper, we have proposed a new robust blind multiuser detector, based on the MOE detector with GSC structure and a QI constraint on the weight vector norm. An optimal VL technique based on a fast low-complexity RSD algorithm is exploited to satisfy the QI constraint. The diagonal loading term is computed using a simple quadratic equation with low complexity. It is evident from the simulations that the new proposed detector outperforms other approaches in terms of the robustness and the steady-state performance. Other attractive merits of the proposed approach are the simplicity and the low computational complexity. Moreover, the adaptive procedure does not require matrix inversion and the algorithm is not sensitive to the improper step-size selection. Future work may include combining RSD-VL algorithm with the channel estimation techniques or with other signature mismatch constraints to produce two-fold robust detectors.

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